GEOMETRY, THE COMMON CORE, AND PROOF

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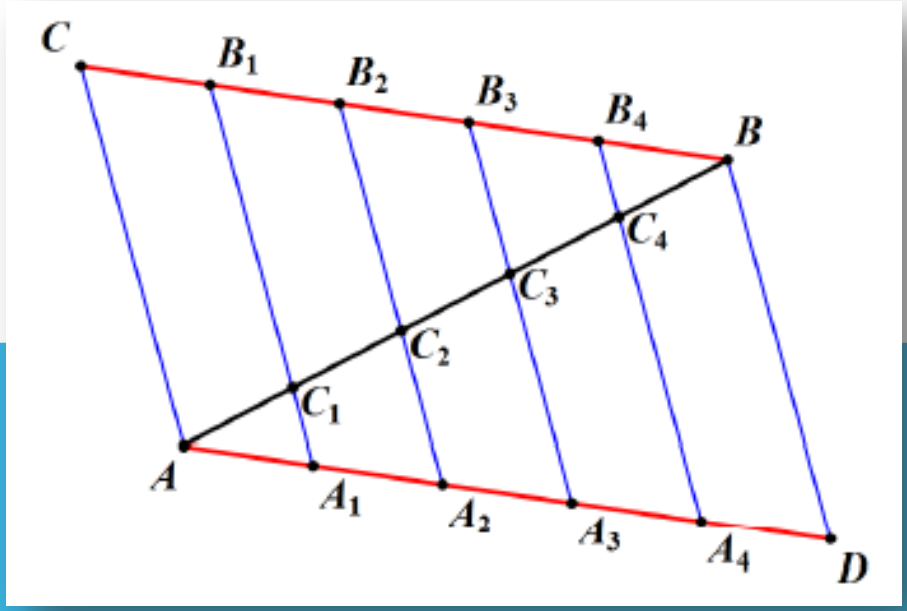
MOTIVATING PROBLEM

Cut a segment into n equal parts, let's say five:

- Patty paper with a segment; lined paper; four points
- Correspondence with last week's construction



HOMEWORK:

- 1. and 2. Importance of exact language
- 2. Importance of exact notation
- 3. Intuition and exact language / definition
- 4. Theorems: tools of convenience (SAS) and understanding (all right angles are congruent)

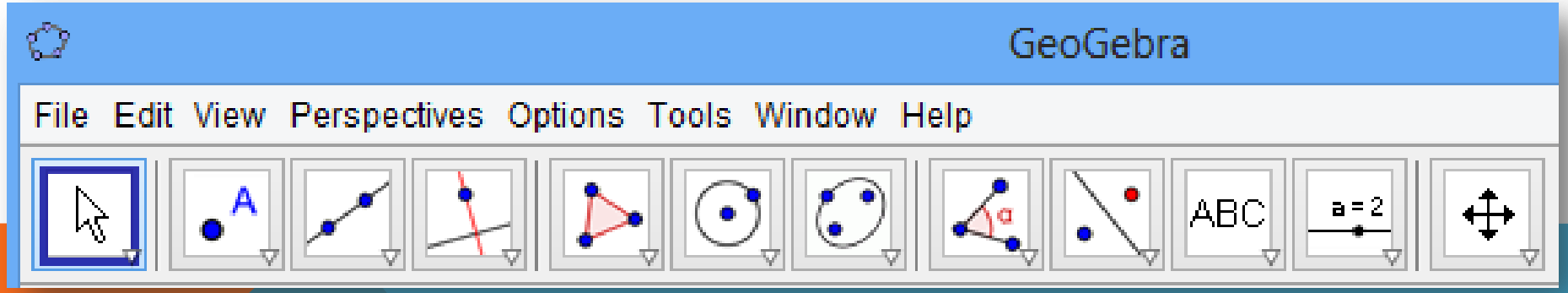
RUSTY COMPASS REVISITED

Rationale for construction; intro to Geogebra

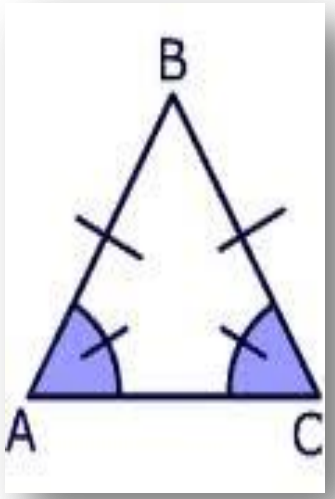
- Basic tools: points, segments, rays, color, thickness, font
- Construction protocol

RIGID MOTION: GEOGEBRA

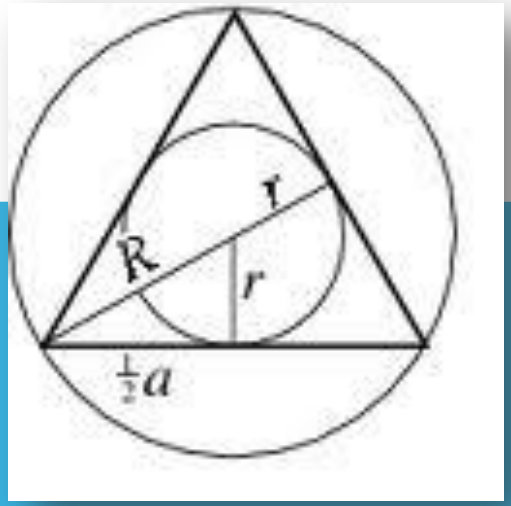
- Translations (along a vector with given direction and length)
- Reflections (in a line)
- Rotations (about a point by an angle



DEFINE USING TRANSFORMATIONS:



- Isosceles triangle
  - Make an isosceles triangle in Geogebra using transformations
- Equilateral/equiangular triangle
  - Make an equilateral triangle in Geogebra using transformations



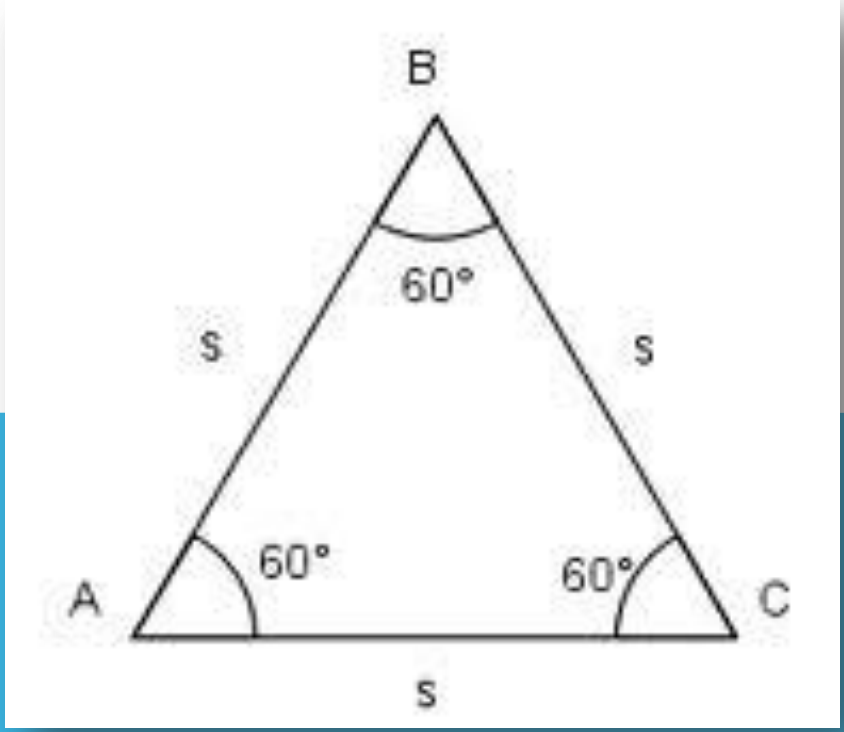
THEOREM 3.22: THE BASE ANGLES OF AN ISOSCELES TRIANGLE ARE CONGRUENT.

- Rewrite Euclid's proof as a two column proof
- Is there a simpler proof using our axioms so far?

EQUILATERAL TRIANGLES

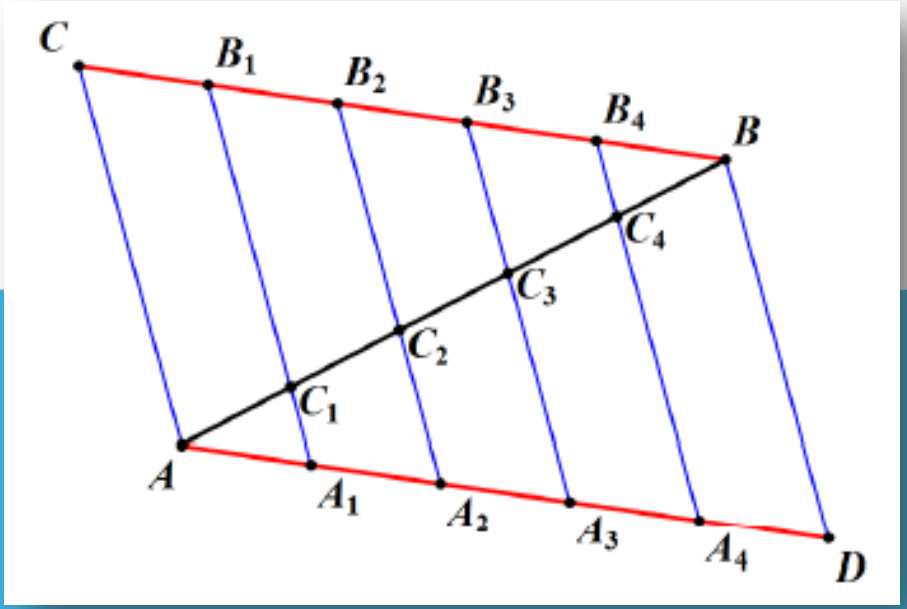
- Prove that the angles are 60 degrees, i.e. 1/6 of a turn (a circle can be cut into 6 equal parts using the radius, i.e. a regular hexagon can be made from six equilateral triangles)

- Prove that equilateral triangles are equiangular



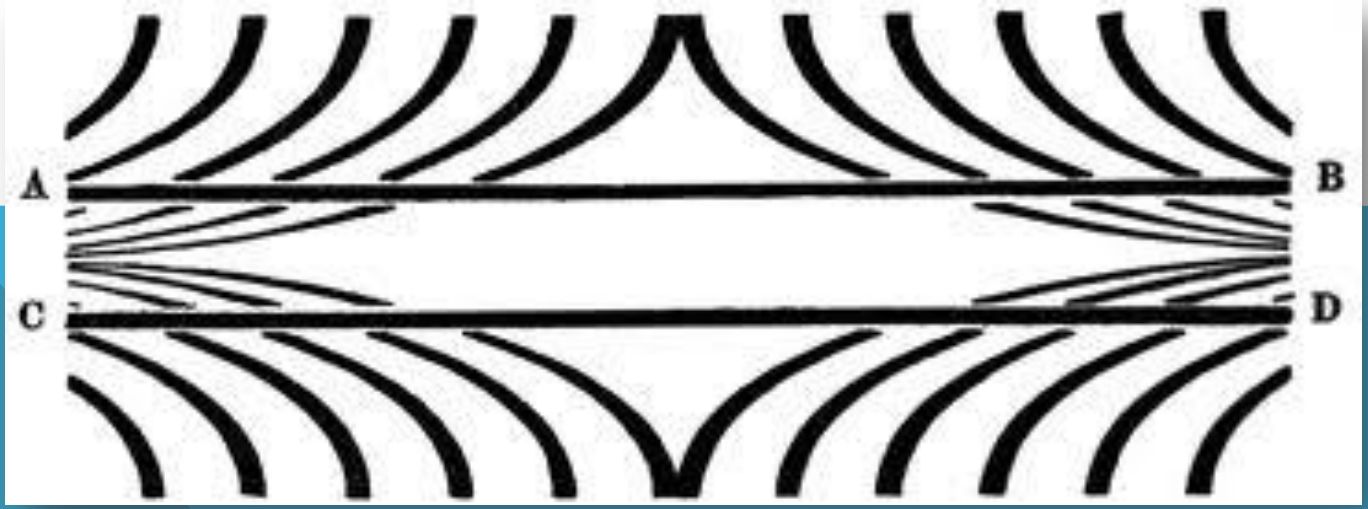
REFOCUS AND REPURPOSE OURSELVES ON THE MOTIVATING PROBLEM

- Prove with any theorems why the construction works
- What concepts/theorems are we using that we need to prove first?
  - Similar triangles and ratios
  - Parallel lines and parallelograms



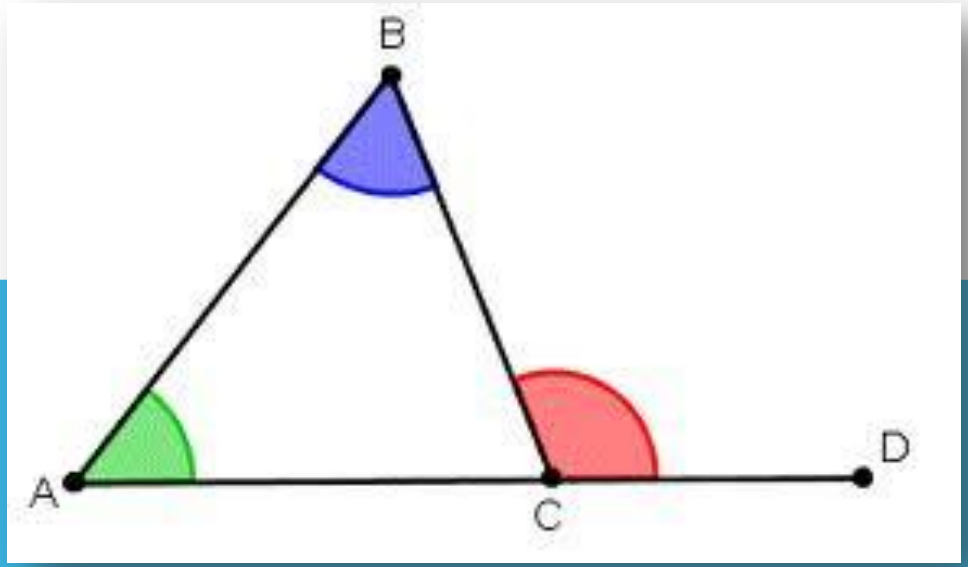
PARALLEL LINE POSTULATE

- What is it?
- Existence vs. Uniqueness
- Euclid



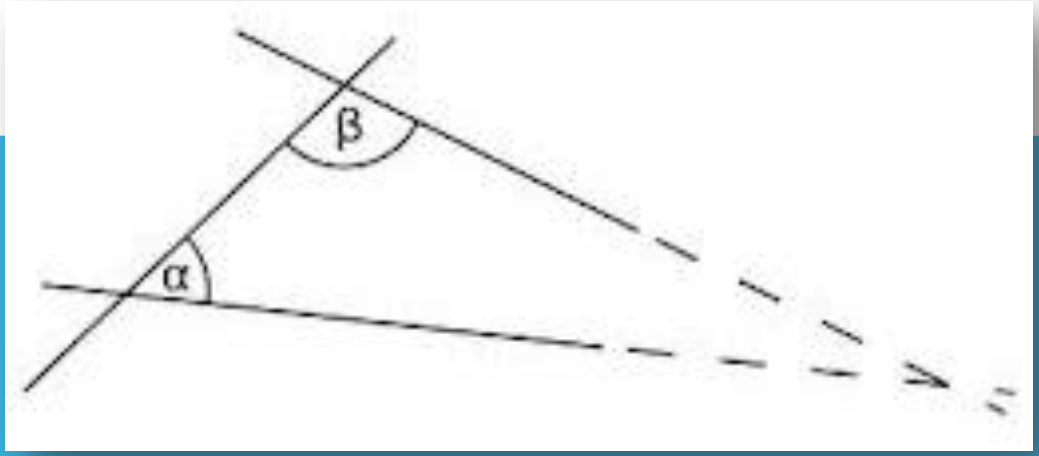
PROPOSITION 16: EXTERIOR ANGLE THEOREM

- Understand Euclid's proof: rewrite in modern English in paragraph form
- Use it to prove the Alternate Interior Angle Theorem (if alternate interior angles are congruent, then the lines are parallel)



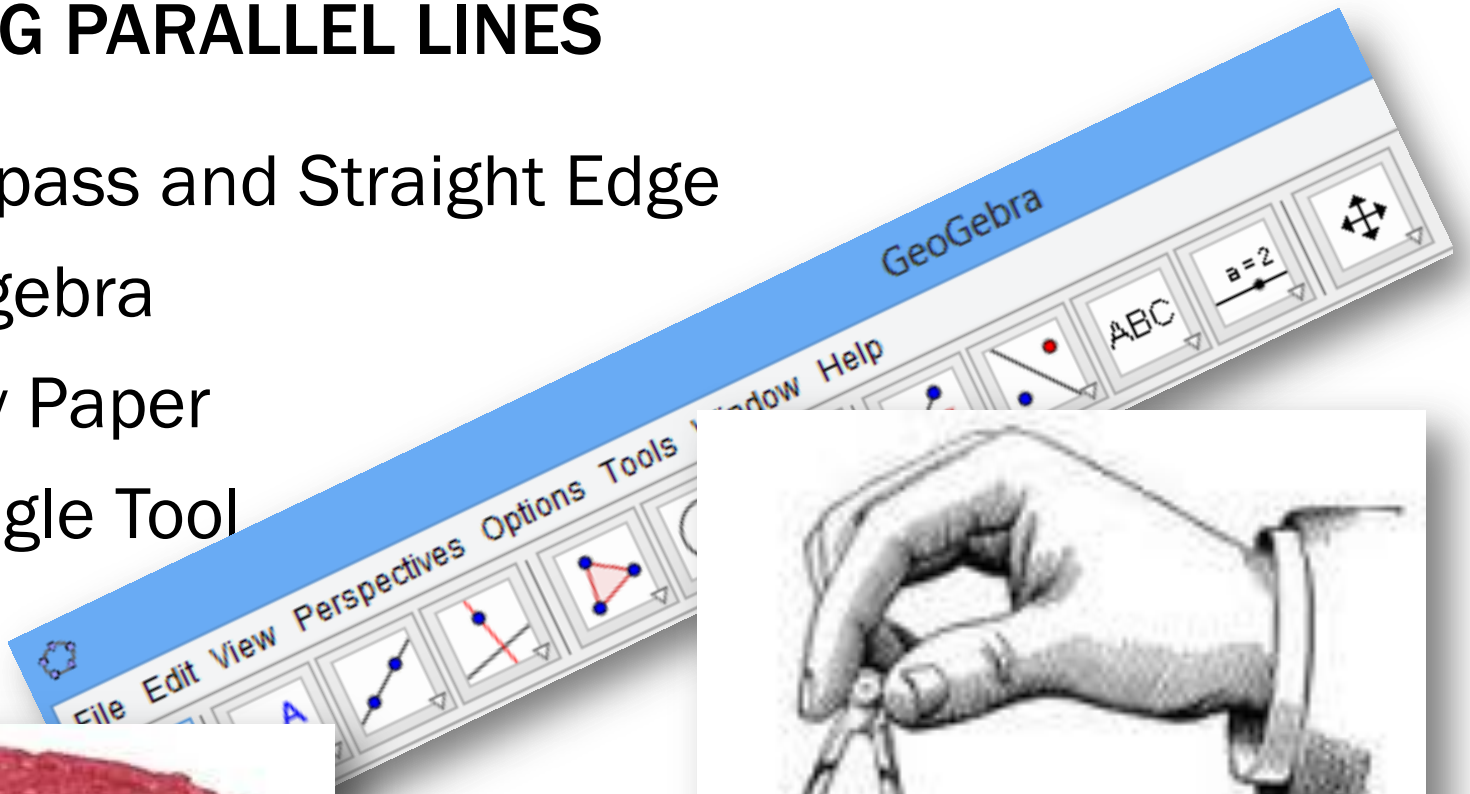
WHY DO WE STILL NEED A PARALLEL LINE POSTULATE?

- Not for existence, but for uniqueness
- Axiom 3.31: If the lines are parallel, then alternate interior angles are equal
  - How does that prove uniqueness?
- State the contrapositive of the above statement
  - Compare it to Euclids “Parallel Line” Postulate



MAKING PARALLEL LINES

- Compass and Straight Edge
- Geogebra
- Patty Paper
- Triangle Tool

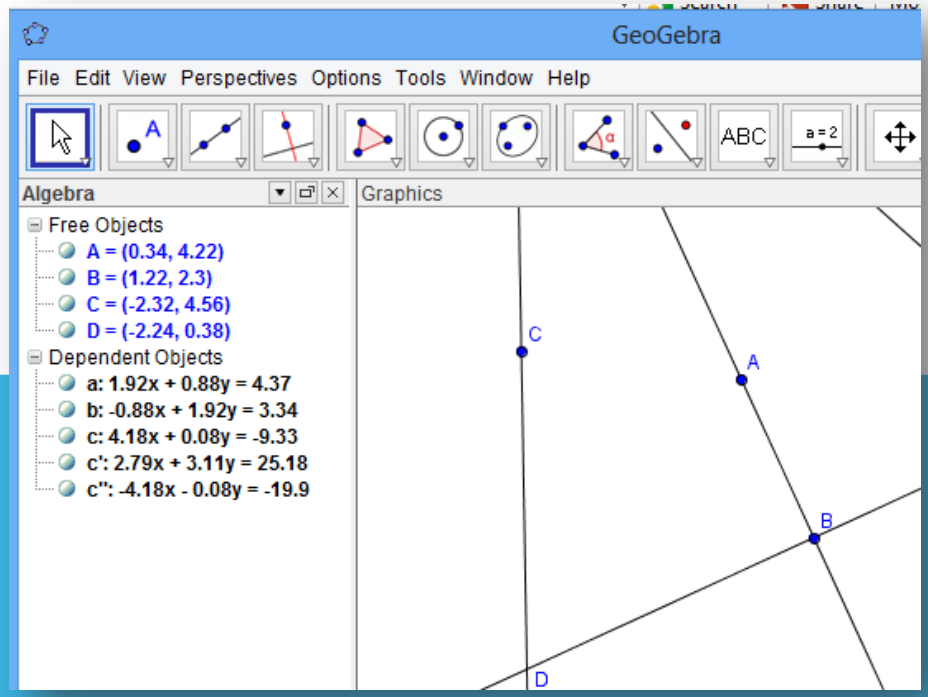


PARALLEL LINES: COMPASS AND STRAIGHT EDGE

- Making perpendicular lines
- Transversal and alternate interior angles
- Transversal and corresponding angles
- Transversal and same-side interior angles

USING GEOGEBRA

- Parallel line command
- Translation command
- Rotation command
- Reflection command



PATTY PAPER

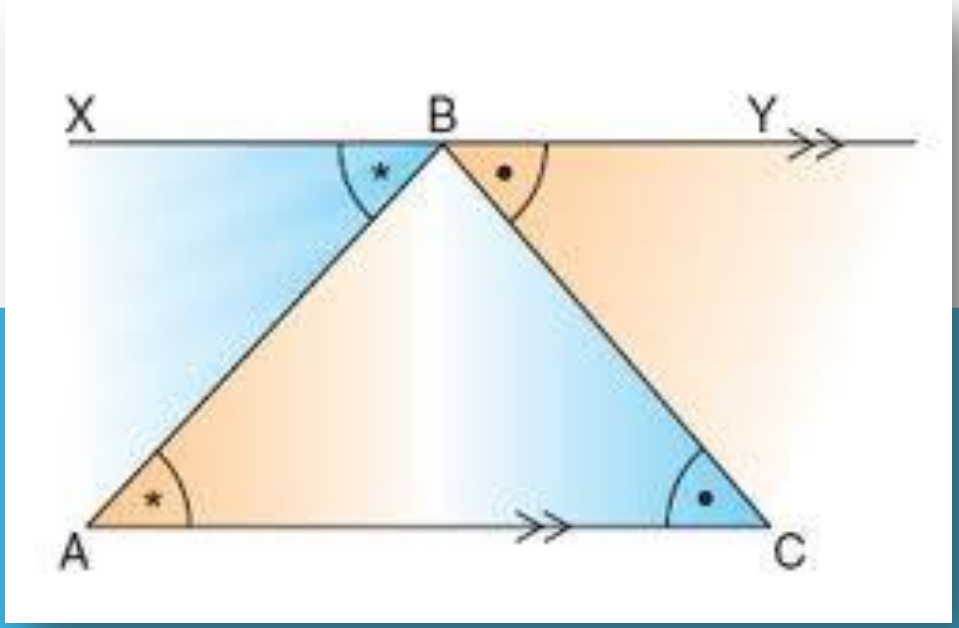
- Perpendicular of a perpendicular
- Rotation
- Reflection

PARALLEL LINE TOOL

- Straight edge and drafting triangle
- Why does it work?

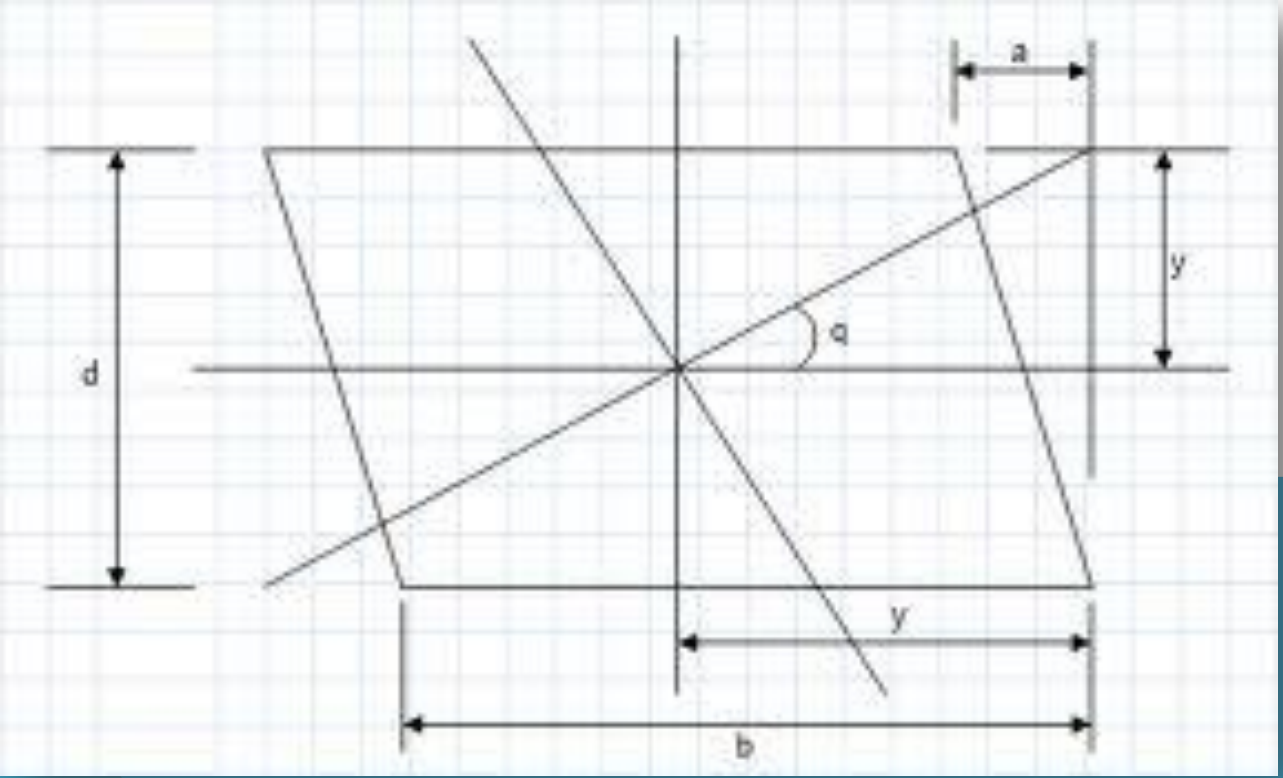
THE ANGLE SUM OF A TRIANGLE

- Prove that the angles of an equilateral/equiangular triangle are 60 degrees, i.e. 1/6 of a circle
- Why do we need the Parallel Line Postulate to prove it?



PARALLELOGRAMS

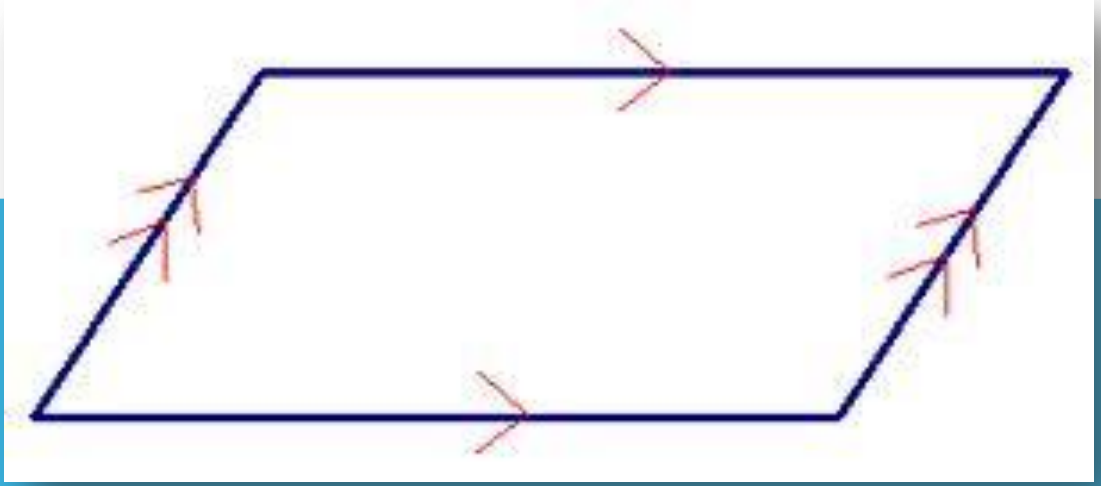
- Definition
- Theorems about parallelograms



PARALLELOGRAMS

Definition

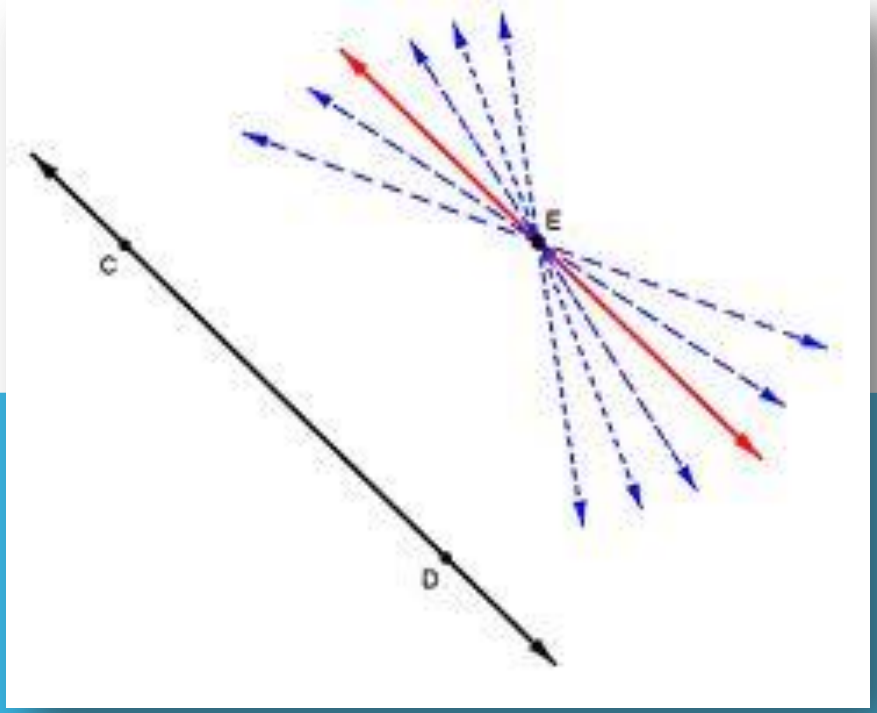
A quadrilateral with two pairs of parallel sides is a parallelogram



PARALLELOGRAMS

Theorems about parallelograms

- Which ones require the Parallel Line Postulate?



WHAT ELSE DO WE NEED?

- Similar Triangles
- Ratios and Rational Numbers
- Multiplication and Division
- Area